Clone with Differentia over infinite alphabet is automatic

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Outline

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1. Introduction
2. Decidability of clone
3. Decidability of differentia
4. Current and futur work

Introduction, ∞

- Set of possible states of a system is finite: its behavior and its specification can be modeled by finite automata
- Specification of infinite behavior: its specification can be modeled by special automata: Büchi, Muller, Rabin
- Domain system is infinite: its verification is in general unreliable
- Requires an extension of finite alphabet automata and regular models:
  - Register and Pebble automata [SF94, KF94, NSV01, Tan10]
  - $\mathfrak{M}$-automata [Bès08]
  - Data automata [BM06, BDM$^+$06]
  - Variable automata [GKS10]

Introduction, ∞, motivations

- Control systems with an infinite data source
- Modeling and|or verification of timed system
- Systems with integer parameters [BHM03]
- Log systems[1] [Via09, BHJS07]
- Semi-structured data and XML documents[2] [CFB$^+$02, BCC$^+$03]

[1] Log systems that store data belonging to an infinite domain.

[2] An XML document can be viewed as a tree whose leaves and branches are usually associated with values belonging to an infinite domain.

Introduction, ∞, motivations

Example (Timed system)

- $s_i \in \Sigma$: finite set of states
- $t_i \in \mathbb{R}^+$ or $\in \mathbb{N}$ discrete or continuous time
- Evolution of the system: word over infinite alphabet $(\Sigma \times \mathbb{R}^+)$ or $(\Sigma \times \mathbb{N})$
- $(s_0, t_0)(s_1, t_1) \dots (s_n, t_n)$

Example (Process management)

- $a_i \in \Sigma$: ensemble fini des actions
- $p_i \in \mathbb{N}$: numro du processus
- Action history: word over infinite alphabet $(\Sigma \times \mathbb{N})$
- $(a_0, p_0)(a_1, p_1) \dots (a_n, p_n)$

Introduction, relational structures, decidability

Definition

A relational structures $\mathfrak{A}$ is a tuple $(A; R_1, R_2, \ldots)$, where A is a non-empty set, called the **domain** of $\mathfrak{A}$, and each $R_i \subseteq A^{r_i}$ is a relation on A with arity r_i.

- $FO(\mathfrak{A})$: all **first-order** statements φ such as $\mathfrak{A} \models \varphi$
- Can we decide whether a statement φ is True or False in $\mathfrak{A}$?

Example

- $FO(\mathbb{N}; =, +)$ is decidable,
- $FO(\mathbb{N}; =, +, \times)$ is not.

Introduction, decidability, classical techniques

- Completeness
- Elimination of quantifiers
- Composition
  - Disjoint union of structures
  - Sum of structures
  - Product of structures
  - Power[3] of a structure
- Interpretation/Reduction
- Automata
  - Automaticity
  - $\mathfrak{M}$-automata [Bès08]

[3] $(\mathbb{N}; \times, =)$ is power of $(\mathbb{N}; +, =)$

Introduction, automaticity

Definition

Let σ be a finite alphabet. A relational structure $\mathfrak{A}$ is **automatic**, if there exists an injective mapping $c : A \longrightarrow \sigma^\star$ such that the images by c of A and all R_i are regular.

- $c(A) = \{c(x) \mid x \in A\}$ is a regular language over $\sigma^\star$.
- Each R_i: $c(R_i) = \{(c(x_1), c(x_2), \ldots, c(x_{r_i})) \mid (x_1, x_2, \ldots, x_{r_i}) \in R_i\}$ is a regular language over $(\sigma^\star)^{r_i}$ too.

Theorem

If a relational structure $\mathfrak{A}$ is automatic then its first-order theory $FO(\mathfrak{A})$ is decidable [BG00].

Introduction, $\mathfrak{M}$-automaticity

In [Bès08] Bès introduced the concept of $\mathfrak{M}$-automata, which is a natural notion of automata for finite words over an infinite alphabet.

Definition

Let Σ denotes an alphabet, finite or not, and let $\mathfrak{M}$ denotes a relational structure with domain Σ. An $\mathfrak{M}$-**automaton** is a finite n-tape synchronous non-deterministic automaton which reads finite words over $\Sigma \cup \{\#\}$ ($\#$ is a padding symbol). Transition rules are triplets of the form (q, φ, q'), where q, q' are states of the automaton, and $\varphi(x_1, \dots, x_n)$ is a first-order formula in the language of $\mathfrak{M}$.

Alexis Bès.

An Application of the Feferman-Vaught Theorem to Automata and Logics for Words over an Infinite Alphabet

Logical Methods in Computer Science, 4(1:8):1–23, 2008.

Introduction, $\mathfrak{M}$-automaticity

Definition

Let $n \geq 1$. A relation $X \subseteq (\Sigma^\star)^n$ is said to be $\mathfrak{M}$-**recognizable** if and only if there exists an $\mathfrak{M}$-automaton $\mathcal{A}$ with n tapes such that $X = L(\mathcal{A})$.

Definition

Let $\mathfrak{M} = (\Sigma; ...)$ and $\mathfrak{N} = (N; R_1, R_2, \dots)$ be two structures. We say that $\mathfrak{N}$ is $\mathfrak{M}$-**automatic** if there exists an injective mapping called coding $c : N \longrightarrow \Sigma^\star$ such that the images by c of N and all R_i are $\mathfrak{M}$-recognizable relations.

Theorem ([Bès08])

Let $\mathfrak{N}$ be a relational $\mathfrak{M}$-automatic structure. If $FO(\mathfrak{M})$ is decidable then $FO(\mathfrak{N})$ is decidable.

Introduction, clone, differentia

- Σ denumerable infinite alphabet
- *finite word* is a finite sequence of symbol (letter) from the alphabet Σ
- Σ^* denotes all finite words over Σ
- Σ^* denotes the integer finite words where $\Sigma = \mathbb{Z}$
- Σ^* denotes the natural finite words where $\Sigma = \mathbb{N}$
- ε denotes the empty word

In the following *word* refers always to *finite word*

Introduction, clone, differentia

Clone

- For a word x of Σ^*, the predicate $clone(x)$ is true, if and only if, x ends with two identical letters

$$clone(x) \Leftrightarrow x = uaa \text{ with } u \in \Sigma^* \text{ and } a \in \Sigma$$

- Shelah [She75] mentions a result of Stupp [Stu75] which was later improved by Muchnik concerning the iteration structure, in which, one define the clone predicate

Differentia

- For a word x of Σ^*, the predicate $diff(x)$ is true, if and only if, all letters of x are different (distinct)

$$diff(x) \Leftrightarrow x = x_1 x_2 \dots x_n \ : \ \forall i \forall j \ : \ i \neq j \Rightarrow x_i \neq x_j$$

Introduction, infinite clone differentia structure, motivation

Nice and interesting properties definable in $(\Sigma^*; \prec, clone, diff, less)$

Example (List of process ID)

$L \in \Sigma^*$ with $L = 3\ 4\ 2\ 1\ 1\ 2\ 0\ 7\ 7$

1. L ends with two different IDs
2. L contains two consecutive identical IDs
3. All IDs in the list L are identical
4. All IDs in the list L are distinct
5. L ends with k identical IDs
6. L contains k consecutive identical IDs
7. L is a decreasing sequence
8. L is an increasing sequence

Introduction, infinite clone differentia structure, motivation

Theorem ([Choffrut et Grigorieff, 2009])

For an infinit alphabet Σ*, the* $\exists\forall\forall$ *theory of the structure* $(\Sigma^*; \prec, \varepsilon, Pred, Eqlast)$ *is undecidable, where:*

- $Pred(a_1 \ldots a_n) = a_1 \ldots a_{n-1}$
- *EqLast denotes the binary relation* $\{(ua, va) \mid u, v \in \Sigma^*, a \in \Sigma\}$*, i.e. the set of pairs of words which end with the same letter*

Christian Choffrut and Serge Grigorieff.
Finite *n*-tape automata over possibly infinite alphabets: Extending a theorem of Eilenberg et al.
Theor. Comput. Sci., 410(1):16–34, 2009.

Outline

Automaticity of $(\Sigma^\star; \prec, clone, less, \sim, \oplus, (R_{p,q})_{p,q\in\Sigma})$

Definition

$(\Sigma^\star; \prec, clone, less, \sim, \oplus, (R_{p,q})_{p,q\in\Sigma})$ denotes the infinite clone structure:

- $\Sigma^\star$ denotes the set of integer finite words, $\Sigma = \mathbb{Z}$.
- $x \prec y$ if and only if x is a strict prefix of y.
- $clone(x) \Leftrightarrow x = uaa$ with $u \in \Sigma^\star$ and $a \in \Sigma$.
- $less(x) \Leftrightarrow x = uab$ with $u \in \Sigma^\star$, $a, b \in \Sigma$ and $a < b$.
- $R_{p,q}(x) \Leftrightarrow x = uab$ with $u \in \Sigma^\star$, $a, b \in \Sigma$ and $|b - a| \equiv q[p]$ avec $p, q \in \Sigma$.
- $x \sim y \Leftrightarrow |x| = |y|$.
- $\oplus(x, y, z) \Leftrightarrow \begin{cases} x = x_1 x_2 \ldots x_i \ldots x_n \text{ and} \\ y = y_1 y_2 \ldots y_i \ldots y_n \text{ and} \\ z = (x_1 + y_1)(x_2 + y_2) \ldots (x_i + y_i) \ldots (x_n + y_n) \end{cases}$

Automaticity of $(\Sigma^\star; \prec, \mathit{clone}, \mathit{less}, \sim, \oplus, (R_{p,q})_{p,q\in\Sigma})$

Example

- $x = [-1, 2, 5]$
- $y = [-1, 2, 5, 2, -3, 4, 4]$
- $x \prec y$, $\mathit{less}(x)$, $\mathit{clone}(y)$ are true.
- $R_{2,1}(x)$ is also true because $|5-2| \equiv 1[2]$.

Example

- $w = [3, 5, 2]$
- $x = [1, 3, 0]$
- $y = [4, 8, 2]$
- $z = [6, 0, 9]$
- $\oplus(w, x, y)$ is true but not $\oplus(x, y, z)$.

Automaticity of $(\Sigma^\star; \prec, clone, less, \sim, \oplus, (R_{p,q})_{p,q\in\Sigma})$

Theorem

The infinite clone structure $(\Sigma^\star; \prec, clone, less, \sim, \oplus, (R_{p,q})_{p,q\in\Sigma})$ *is* $\mathfrak{Z}$*-automatic, with* $\mathfrak{Z} = (\Sigma; 0, <, +)$.

Proof.

Let $\Sigma = \mathbb{Z}$. To prove that the infinite clone structure is $\mathfrak{Z}$-automatic, we use a coding d, that establish the $\mathfrak{Z}$-automaticity of the structure's domain and the $\mathfrak{Z}$-automaticity of the structure's predicates. This coding returns the differential of each word in $\Sigma^\star$. $\square$

Definition

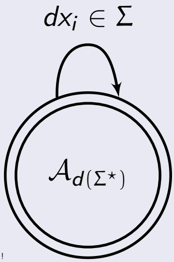
Let $x = x_1x_2 \dots x_n \in \Sigma^\star$ an integer word of length n. We call **differential** of x the integer word denoted by $d(x) = x_1 d_2 d_3 \dots d_n \in \Sigma^\star$ where $d_k = x_k - x_{k-1}$. By convention $d(\varepsilon) = \varepsilon$.

Example

- $x = [-1, 5, 2, 2, -3, 4, 4]$
- $d(x) = [-1, 6, -3, 0, -5, 7, 0]$

Automaticity of $(\Sigma^\star; \prec, clone, less, \sim, \oplus, (R_{p,q})_{p,q \in \Sigma})$

Proof.



Domain's codes $d(\Sigma^\star)$ is $\mathfrak{Z}$-automatic. $\mathcal{A}_{d(\Sigma^\star)}$ is a $\mathfrak{Z}$-automaton that recognizes $d(\Sigma^\star)$ all codes over $\Sigma^\star$.

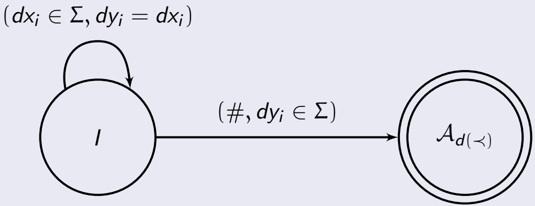
$\square$

Example

- $x = [-1, 5, 2, 2, -3, 4, 4]$
- $d(x) = [-1, 6, -3, 0, -5, 7, 0]$

Automaticity of $(\Sigma^\star; \prec, \mathit{clone}, \mathit{less}, \sim, \oplus, (R_{p,q})_{p,q\in\Sigma})$

Proof.



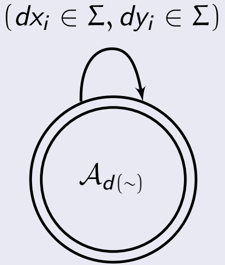
Prefix predicate's codes, $d(\prec)$, is $\mathfrak{Z}$-automatic. $x, y \in \Sigma^\star : x \prec y \Leftrightarrow d(x) \prec d(y)$. The $\mathfrak{Z}$-automaton $\mathcal{A}_{d(\prec)}$ is able to verify whether a word is prefix of another. □

Example

$$x = [1, 5, 7, 3, 4, 6] \qquad d(x) = [1, 4, 2, -4, 1, 2]$$
$$y = [1, 5, 7] \qquad d(y) = [1, 4, 2]$$

Automaticity of $(\Sigma^\star; \prec, clone, less, \sim, \oplus, (R_{p,q})_{p,q\in\Sigma})$

Proof.



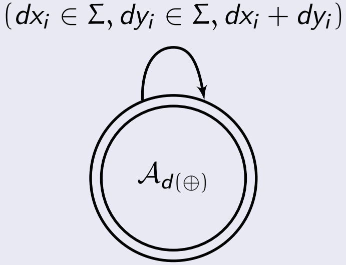
Equal length predicate's codes, $d(\sim)$, is $\mathfrak{Z}$-automatic. It is easy to verify that two words have the same length if and only if the code of the first and the second have the same length. For $x, y \in \Sigma^\star$, we have $x \sim y \Leftrightarrow d(x) \sim d(y)$. The $\mathfrak{Z}$-automaton $\mathcal{A}_{d(\sim)}$ is able to verify whether two words have the same length.

□

Example

- $x = [1, 5, 7]$ $d(x) = [1, 4, 2]$
- $y = [3, 4, 6]$ $d(y) = [3, 1, 2]$

Proof.



Sum letter by letter relation's codes, $d(\oplus)$, is also $\mathfrak{Z}$-automatic. $\mathcal{A}_{d(\oplus)}$ is a $\mathfrak{Z}$-automaton able to do the sum letter by letter. For $x, y, z \in \Sigma$, we have $\oplus(x, y, z) \Leftrightarrow \oplus(d(x), d(y), d(z))$.

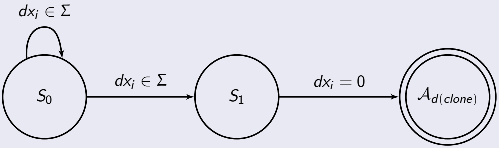
$\square$

Example

- $x = [1, 3, 5]$ $d(x) = [1, 2, 2]$
- $y = [2, 4, 4]$ $d(y) = [2, 2, 0]$
- $z = [3, 7, 9]$ $d(z) = [3, 4, 2]$

Automaticity of $(\Sigma^\star; \prec, clone, less, \sim, \oplus, (R_{p,q})_{p,q\in\Sigma})$

Proof.

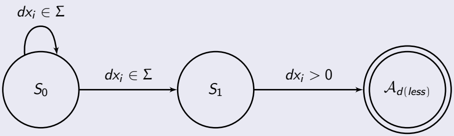


Clone predicate's codes, $d(clone)$, is $\mathfrak{Z}$-automatic. $clone(x) \Leftrightarrow d(x)$ ends with 0. $\mathcal{A}_{d(clone)}$ is a $\mathfrak{Z}$-automaton that checks the clone property. □

Example

- $x = [1, 5, 7, 3, 4, 4]$
- $d(x) = [1, 4, 2, -4, 1, 0]$

Proof.



Less predicate's codes, $d(less)$, is $\mathfrak{Z}$-automatic.

$less(x) \Leftrightarrow d(x)$ ends with $a > 0$.

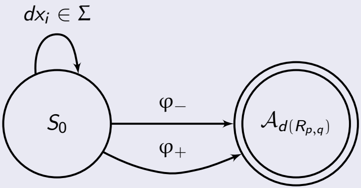
$\mathcal{A}_{d(less)}$ is a $\mathfrak{Z}$-automaton that checks if the code of an integer word ends with a positive integer. □

Example

- $x = [1, 5, 7, 3, 4, 14]$
- $d(x) = [1, 4, 2, -4, 1, 10]$

Automaticity of $(\Sigma^\star; \prec, \mathit{clone}, \mathit{less}, \sim, \oplus, (R_{p,q})_{p,q\in\Sigma})$

Proof.



$$\varphi_+ : \quad dx_i \geq 0 \wedge \exists N \geq 0 \; \underbrace{N + N + \cdots + N}_{\underbrace{p}_{\mathit{constant}} \; \mathit{times}} + \underbrace{q}_{\mathit{constant}} = dx_i$$

$$\varphi_- : \quad dx_i < 0 \wedge \exists N \geq 0 \; \underbrace{N + N + \cdots + N}_{\underbrace{p}_{\mathit{constant}} \; \mathit{times}} + \underbrace{q}_{\mathit{constant}} + dx_i = 0$$

For $p, q \in \Sigma$, $d(R_{p,q})$, is $\mathfrak{Z}$-automatic.
The following $\mathfrak{Z}$-automaton is able to check the $R_{p,q}$ predicate. $\square$

Automaticity of $(\Sigma^\star; \prec, \mathit{clone}, \mathit{less}, \sim, \oplus, (R_{p,q})_{p,q\in\Sigma})$

Corollary

The first-order theory of the infinite clone structure $(\Sigma^\star; \prec, \mathit{clone}, \mathit{less}, \sim, \oplus, (R_{p,q})_{p,q\in\Sigma})$ is decidable.

Proof.

Direct consequence of the $\mathfrak{Z}$-automaticity of a structure is the decidability of its first-order theory. $\square$

Outline

Automaticity of $(\Sigma^\star; \prec, clone, less, \sim, \oplus, (R_{p,q})_{p,q\in\Sigma})$

Definition

$(\Sigma^\star; \prec, clone, diff)$ denotes the infinite clone differentia structure, where:

- $\Sigma^\star$ denotes the set of natural finite words, $\Sigma = \mathbb{N}$.
- $x \prec y$ if and only if x is a strict prefix of y.
- $clone(x)$ is true if and only if x ends with two identical letters: $clone(x) \Leftrightarrow x = uaa$ with $u \in \Sigma^\star$ and $a \in \Sigma$.
- $diff(x)$ is true if and only if all letters of x are different (distinct): $diff(x) \Leftrightarrow x = x_1 x_2 \ldots x_n$ with $x_1, x_2, \ldots, x_n \in \Sigma$ and $\forall i \forall j$ $i \neq j \Rightarrow x_i \neq x_j$.

Automaticity of $(\Sigma^{\star}; \prec, \mathit{clone}, \mathit{diff})$

Example

- $x = [7, 2, 5]$
- $y = [7, 2, 5, 2, 3, 4, 4]$
- $x \prec y$, $\mathit{clone}(y)$ are true.
- $\mathit{diff}(x)$ is true but not $\mathit{diff}(y)$.

Automaticity of $(\Sigma^\star; \prec, \mathit{clone}, \mathit{diff})$

Theorem

The infinite clone differentia structure, $(\Sigma^\star; \prec, \mathit{clone}, \mathit{diff})$, is automatic.

Proof.

Let $\Sigma = \mathbb{N}$. To prove that the infinite clone differentia structure is automatic, we define a coding ν which establishes the rationality of the structure's domain, of the prefix, of the *clone* and of the differentia predicate. $\square$

Basic idea...

1. Let $D = \{x \in \Sigma^\star \mid \mathit{diff}(x)\}$
2. Any natural word can be divided into sub words of D.
3. Any word of D can be codded by a natural word and vice versa.

Automaticity of $(\Sigma^\star; \prec, \mathit{clone}, \mathit{diff})$

Definition

The function s divides a natural word into a sequence of sub words of D.

$$\begin{array}{rcl} s : \Sigma^\star & \longrightarrow & D^\star \\ x = x_1 x_2 \ldots x_n & \longmapsto & s(x) = u_1 u_2 \ldots u_m \end{array}$$

- $m \leq n$
- a word in D stays the same under the function s, $x \in D \Leftrightarrow s(x) = x$

Example

$$[3,0,6,0,1,2,5,2,4,5,5,5,3,4,8,9] \xrightarrow{s} [3,0,6][0,1,2,5][2,4,5][5][5,3,4,8,9]$$

$$[5,2,6,8,3,6,4,8,5,4,4,4,5,6,8,5] \xrightarrow{s} [5,2,6,8,3][6,4,8,5][4][4][4,5,6,8][5]$$

$$[1,3,5,7,8,0,9,6,4,2] \xrightarrow{s} [1,3,5,7,8,0,9,6,4,2] \text{ one sub word}$$

Definition

For an ordered set $S = \{x_1 < x_2 < \cdots < x_i < \ldots\} \subseteq \Sigma$, we denote by $I_S^{x_i}$ the index of the element x_i in S. We have $I_S^{x_i} = i$.

We denote by δ the bijection which encodes every word in D by a natural word in $(\Sigma - \{0\})^\star$ where:

$$\begin{array}{rcl} \delta : D & \longrightarrow & (\Sigma - \{0\})^\star \\ x = x_1 x_2 \ldots x_i \ldots x_n & \longmapsto & \delta(x) = y_1 y_2 \ldots y_i \ldots y_n \\ y_i & = & I^{x_i}_{\Sigma - \{x_1, x_2, \ldots, x_{i-1}\}} \end{array}$$

Automaticity of $(\Sigma^\star; \prec, \mathit{clone}, \mathit{diff})$

Example

Let $x = [5, 3, 4, 2, 1]$. Then x is a natural word whose letters are distinct. We can verify that :

$$\begin{aligned}
\delta(x) &= \delta([5,3,4,2,1]) \\
&= [I^5_{\{0,1,2,3,4,5,\dots\}}, I^3_{\{0,1,2,3,4,\dots\}}, I^4_{\{0,1,2,4,\dots\}}, I^2_{\{0,1,2,3,\dots\}}, I^1_{\{0,1,3,\dots\}}] \\
&= [6,4,4,3,2]
\end{aligned}$$

Automaticity of $(\Sigma^\star; \prec, \mathit{clone}, \mathit{diff})$

Example

Inversely, we can get x from $[6, 4, 4, 3, 2]$:

- the 6^{th} element in $\Sigma = \{0, 1, 2, 3, 4, 5, 6, \ldots\}$ is $5 = x_1$
- the 4^{th} element in $\Sigma - \{x_1\} = \{0, 1, 2, 3, 4, 6, \ldots\}$ is $3 = x_2$
- the 4^{th} element in $\Sigma - \{x_1, x_2\} = \{0, 1, 2, 4, 6, \ldots\}$ is $4 = x_3$
- the 3^{rd} element in $\Sigma - \{x_1, x_2, x_3\} = \{0, 1, 2, 6, \ldots\}$ is $2 = x_4$
- the 2^{nd} element in $\Sigma - \{x_1, x_2, x_3, x_4\} = \{0, 1, 6, \ldots\}$ is $1 = x_5$

So $\delta^{-1}([6, 4, 4, 3, 2]) = [5, 3, 4, 2, 1]$

Automaticity of $(\Sigma^\star; \prec, \mathit{clone}, \mathit{diff})$

Example

$$\nu : \Sigma^\star \longrightarrow \{a, b, c\}^\star$$

$$\xrightarrow{\Sigma} \quad [5, 2, 6, 8, 3, 6, 4, 8, 5, 4, 4, 4, 5, 6, 8, 5]$$

$$\xrightarrow{s} \quad [5, 2, 6, 8, 3][6, 4, 8, 5][4][4][4, 5, 6, 8][5]$$

$$\xrightarrow{\delta} \quad [6, 3, 5, 6, 3][7, 5, 7, 5][5][5][5, 5, 5, 6][6]$$

$$\xrightarrow{-/+} \quad [+6, +3, -5, +6, +3][+7, -5, +7, +5][-5][-5][+5, -5, +5, +6][+6]$$

$$\xrightarrow{\mathit{separator}} \quad [+6, +3, -5, +6, +3, +0, -5, +7, +5, -0, -0, +0, -5, +5, +6, +0]$$

$$\xrightarrow{\{a,b,c\}} \quad [a^6 ba^3 ba^5 ca^6 ba^3 bba^5 ca^7 ba^5 bccba^5 ca^5 ba^6 bb]$$

Automaticity of $(\Sigma^\star; \prec, \mathit{clone}, \mathit{diff})$

Lemma

Universe's codes $\nu(\Sigma^\star)$ is regular.

Proof.

A word on $\{a, b, c\}$ corresponds to a code via ν:

- a sequence of (positive letters, a negative letter, positive letters, zero)
- a sequence of only positive letters

$$\nu(\Sigma^\star) \quad = \quad (P^\star NP^\star(b+c))^\star P^\star NP^\star bP^\star + P^\star \text{ where } P = a^+ b \ , \ N = a^+ c$$

$\square$

Example

$[5, 2, 6, 8, 3, 6, 4, 8, 5, 4, 4, 4, 5, 6, 8, 5]$

$[+6, +3, -5, +6, +3, +0, -5, +7, +5, -0, -0, +0, -5, +5, +6, +0]$

Lemma

Clone predicate's codes $\nu(\mathit{clone})$ is regular.

Proof.

The code of every non-empty *clone* word in $\Sigma^\star$ is a ternary word that ends with *cb*.

$$\mathit{clone}(x) \quad \Leftrightarrow \quad \nu(x) \in \nu(\Sigma^\star) \cap \{a, b, c\}^\star cb$$

□

Example

$[5, 2, 6, 8, 3, 6, 4, 8, 5, 4, 4]$

$[+6, +3, -5, +6, +3, +0, -5, +7, +5, -0, +0]$

$[a^6 ba^3 ba^5 ca^6 ba^3 bba^5 ca^7 ba^5 bcb]$

Automaticity of $(\Sigma^\star; \prec, clone, diff)$

Lemma

Differentia predicate's codes $\nu(diff)$ is regular.

Proof.

The code of every non-empty *diff* word in $\Sigma^\star$ is a sequence of only positive letters.

$$\nu(diff) = \nu(D) = P^\star = (a^+ b)^\star$$

$\square$

Example

$[5, 2, 6, 8, 3]$

$[+6, +3, +5, +6, +3]$

$[a^6 ba^3 ba^5 ca^6 ba^3 b]$

Automaticity of $(\Sigma^{\star}; \prec, \mathit{clone}, \mathit{diff})$

Corollary

The first-order theory of the infinite clone differentia structure $FO(\mathfrak{D})$ is decidable.

Proof.

Decidability of the first-order theory of the infinite clone differentia structure is a direct consequence of its automaticity. $\square$

Outline

Current and futur work

- Automaticity and decidability of some structures theory over countable infinite alphabet
- Developing most advanced coding that maintain the decidability and the automaticity of structures while adding predicates more complex and expressive, such as the predicate $diff_k$
- Studying the complexity
- Identify appropriate models of automata in order to identify everything that can be defined in such theory

Marco Brambilla, Stefano Ceri, Sara Comai, Piero Fraternali, and Ioana Manolescu.
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