

# Optimization on multidimensional hierarchies

ESIEE LIGM A3SI

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Journee

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# Outline

- Thesis outline
  - Objectives
  - Domain of Application
- Geographic Information Systems
  - Data Sources and Features
- Optimization on Hierarchies
  - Theory
  - Examples

# Objectives

- Multivariable optimization of pyramid of segmentations by Mathematical Morphology
- Mathematical Morphology for GIS type data
- Application to model the evolution of PACA(Provence-Alpes-Côte d'Azur)

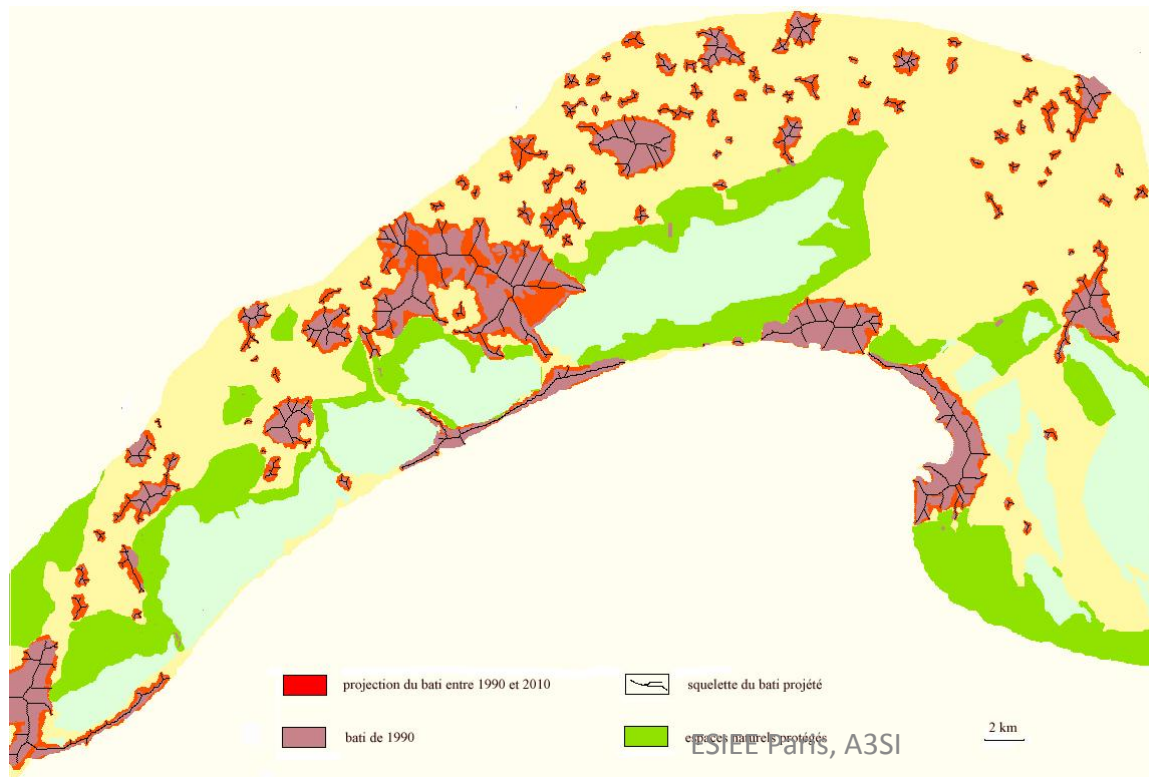
# Domain of Application

- Water resources modeling in Region of PACA (Provence-Alpes-Côte d'Azur)
  - Evolution in Population (Usage of water in swimming pools, apartments, residences)
  - Usage of different types of agriculture methods (to optimize water usage)
  - Predictive Model for utilization of water resources



# Domain of Application

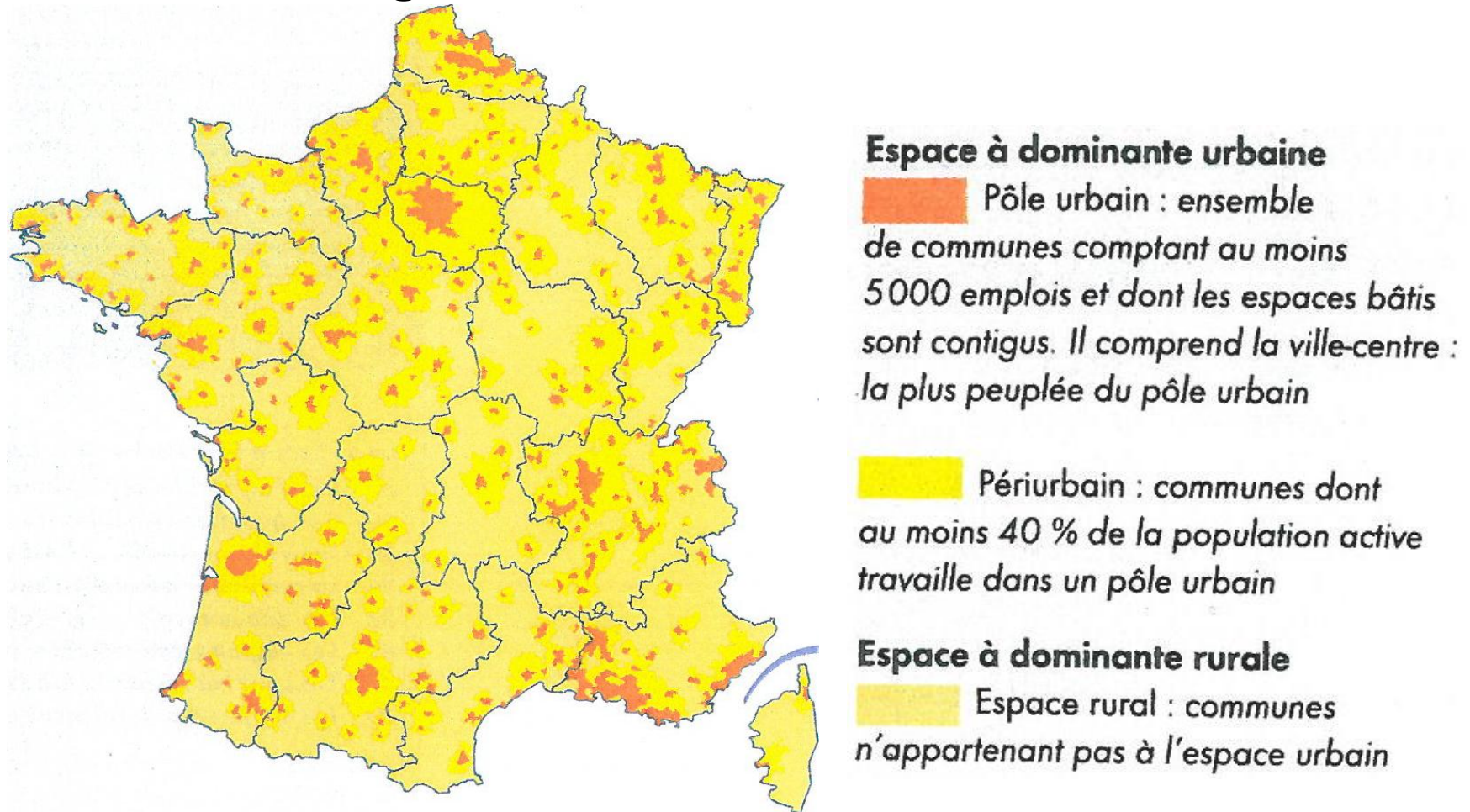
- Modeling the evolution of population
  - Modeling City growths, expansion, reduction
  - Modeling Suburb Evolution



An example Forecast  
of the expansion of  
Montpellier

# Domain of Application

## Modeling Suburb Evolution

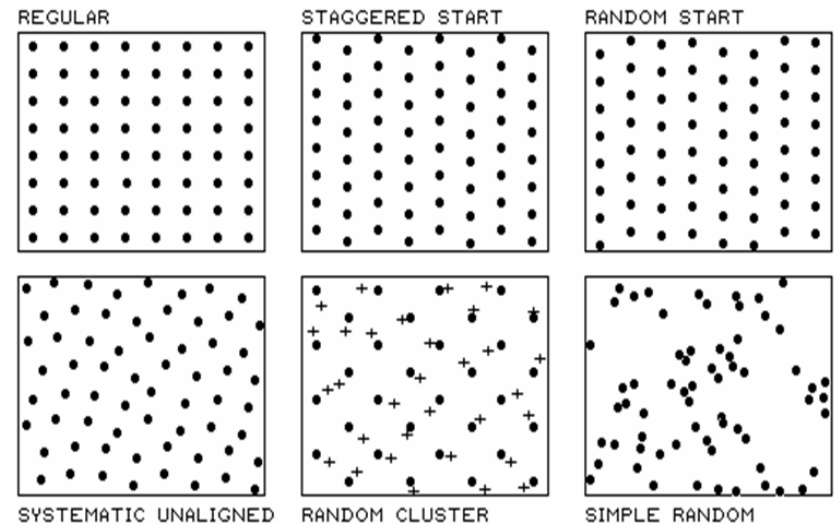
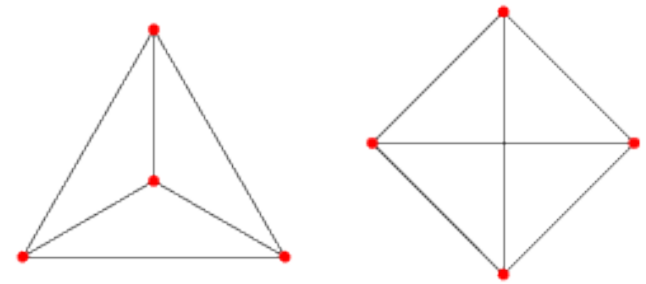




# GEOSPATIAL DATA

# Types of Geospatial Data

- Four Data Features:
  - Planar Graph (aggregated data  
- communes, departments  
...Eg: population)
  - Point-wise Geographical  
Information (Altitude Map,...)
  - Multivariable Sources  
(Population, Income, Water  
resources ...)
  - Time Series ( Population  
Censuses, and other time  
series)

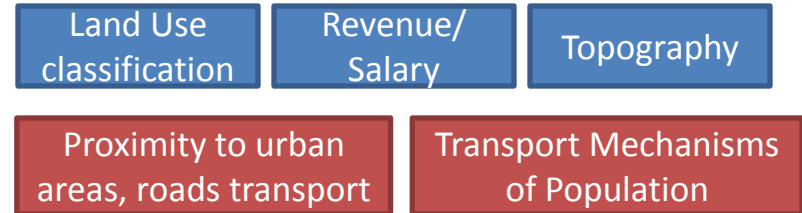




# Types of Geospatial Data

- Four Data Features:

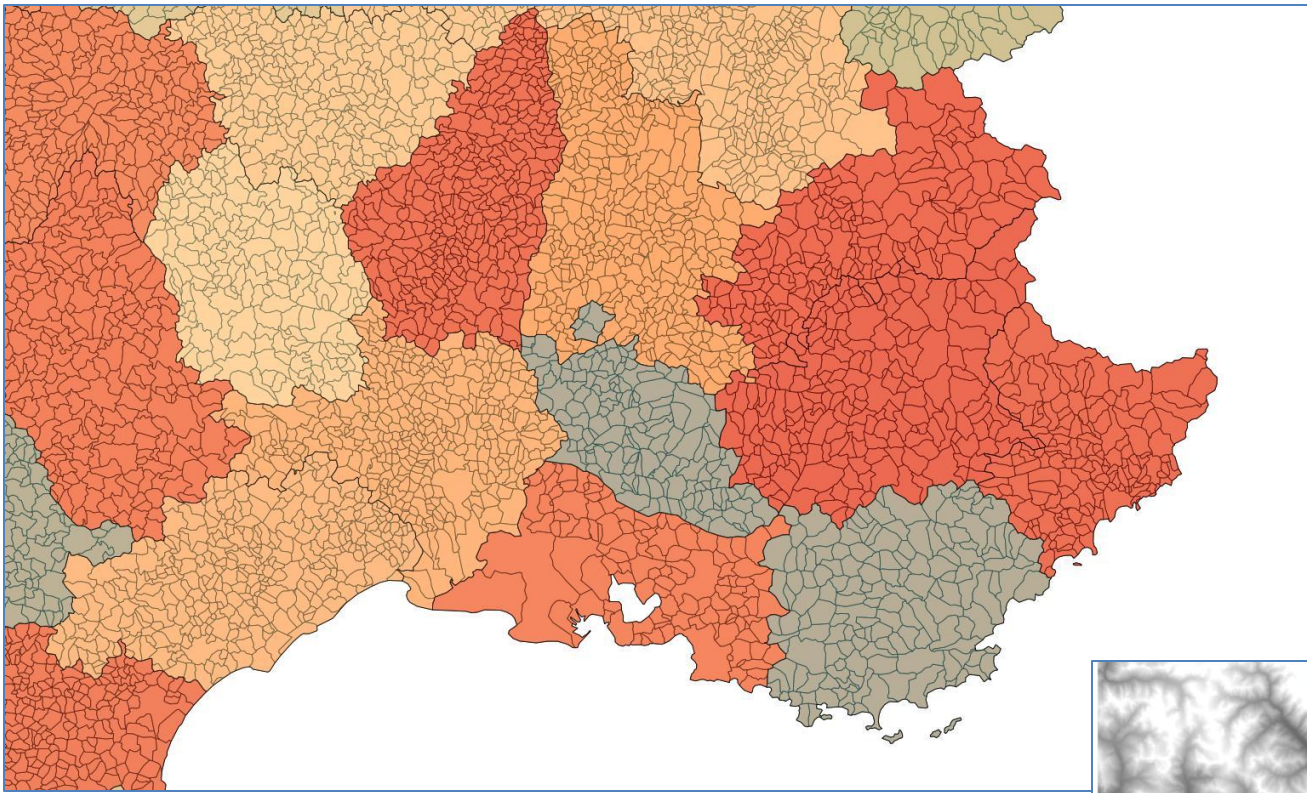
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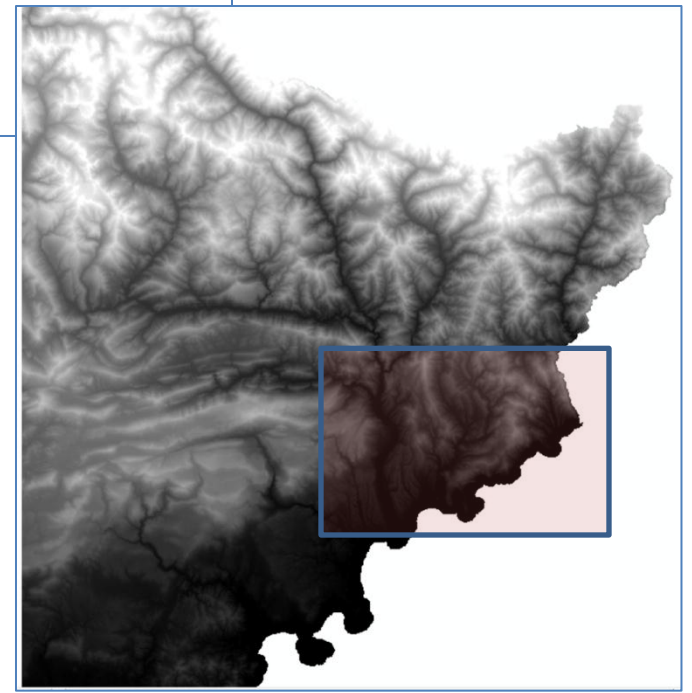
Space partitioning



Spatio-Temporal Series



Communes in PACA (Planar Graph)  
Relief of the zone in PACA (Pixel Data)



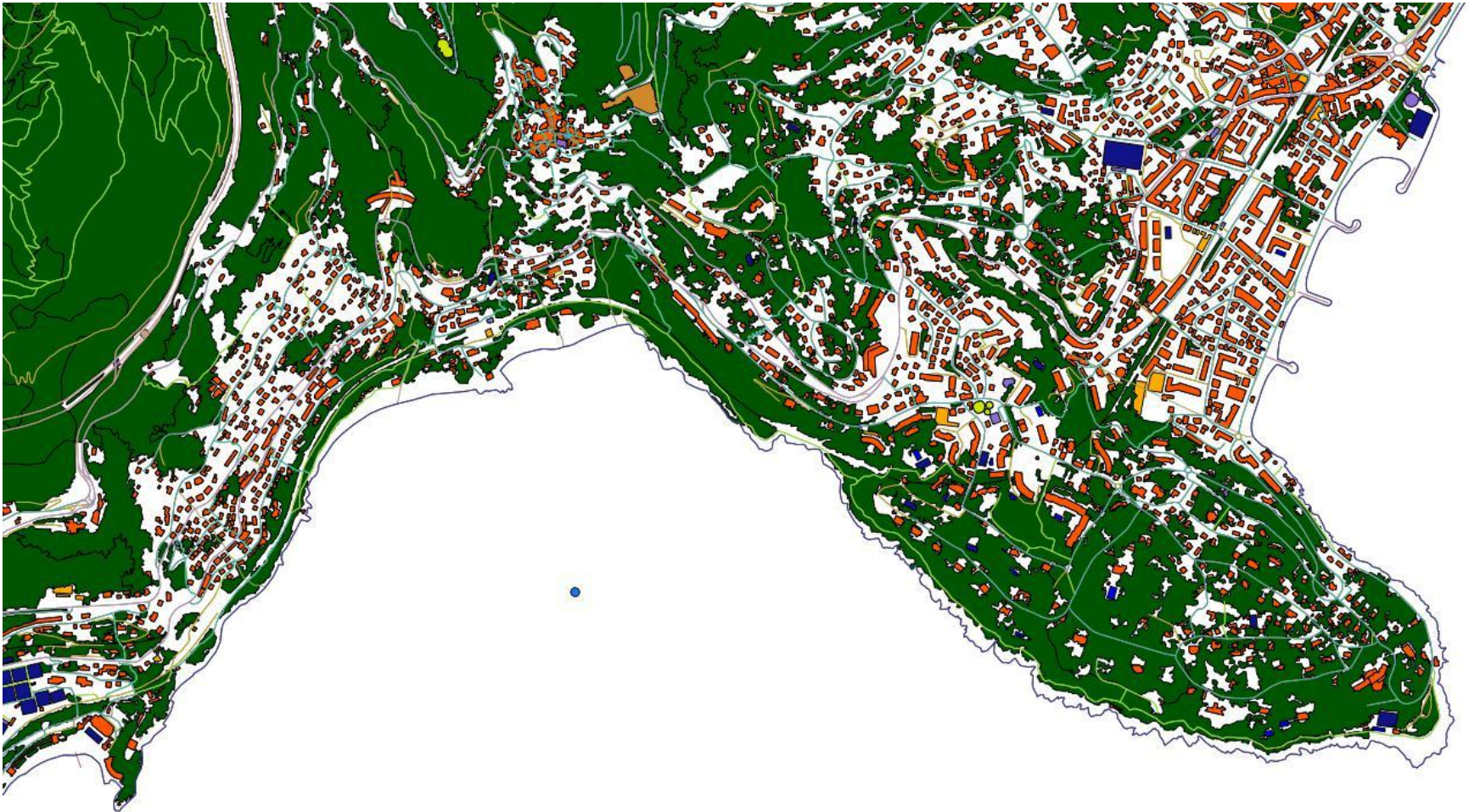
# Nature of Multivariable Data

DATASET TITLE	Type of data	Comments
Population	- per 200x200 meter sq tile - per 1x1 km sq tile	Data is in the form of <x y value> raster
Population	- per commune/dept/region/	data is a list <commune value> - Time series 2008, 1999, 1990, 1982, 1975, 1968
Revenue Salaries	- per commune	data is a list <unit (comm/dept) value> - Time series 2005,2006,2007,2008,2009 Source: <C>

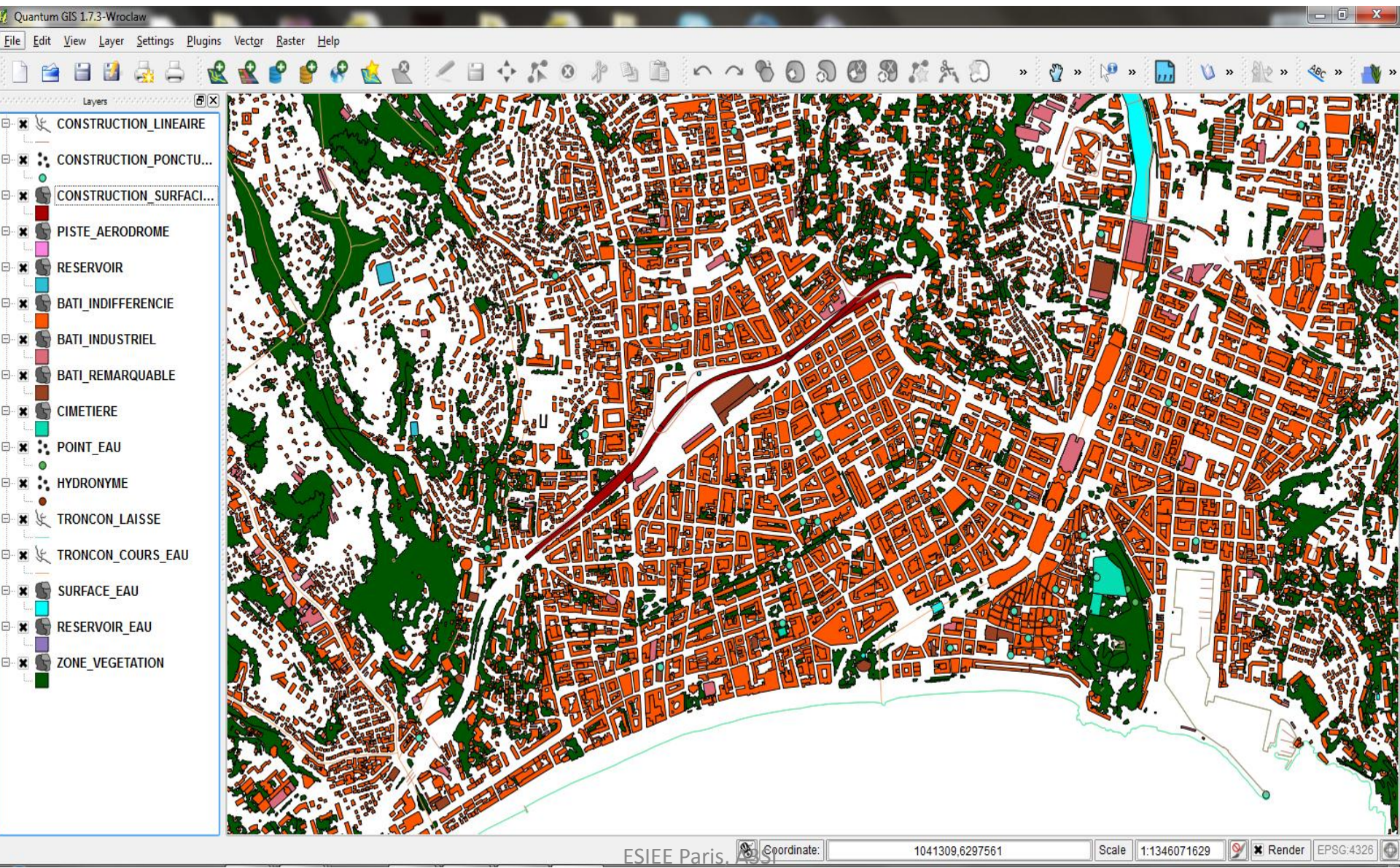
# Nature of Multivariable Data

Administrative boundaries	Data contains indexed communes/cantons/arrondissement/depts with their boundaries in XY locations	data contains various values: <commune idx [xy] centroids of polygons, [xy] of polygon>
Altitude	- per 50x50 meter sq tile	<x y altitude> Altitude sampling is not registered with population sampling
Other Maps	- Vector file defined per department	Equipment, Occupation Sol, Hydrographie, Toponymie, road/path/routes,

# Classified Data Land Use in PACA



# Classified Data - Land Use in PACA



# GIS Software tools

- ArcGIS, QGIS (since the 1980s)
- Problems in having well “registered” maps
- Coordinate systems and Projection systems?
- Data types and Datastructures (MIF, MID ...)



# Processing Geospatial Data

- For one spatial variable, two steps are performed
  - Image Segmentation
  - Pyramid of Segmentations (captures scale)
- Segmentation is performed to delineate
  - Zones in which a given criterion is uniform
  - Border between uniform zones
- This results in a **partition** of the space



# PYRAMIDS

# Generation of a hierarchy of partitions

The input image is represented as a Edge weighted graph

One calculates a sequence of minima by successive floodings.

At each step a watershed is created

The partitions of the vertices increase so that watershed lines are associated with a saliency

Therefore the saliency is a compact representation of the whole hierarchy.

# Input Image

$(R+G+B)/3$  -  
Luminance

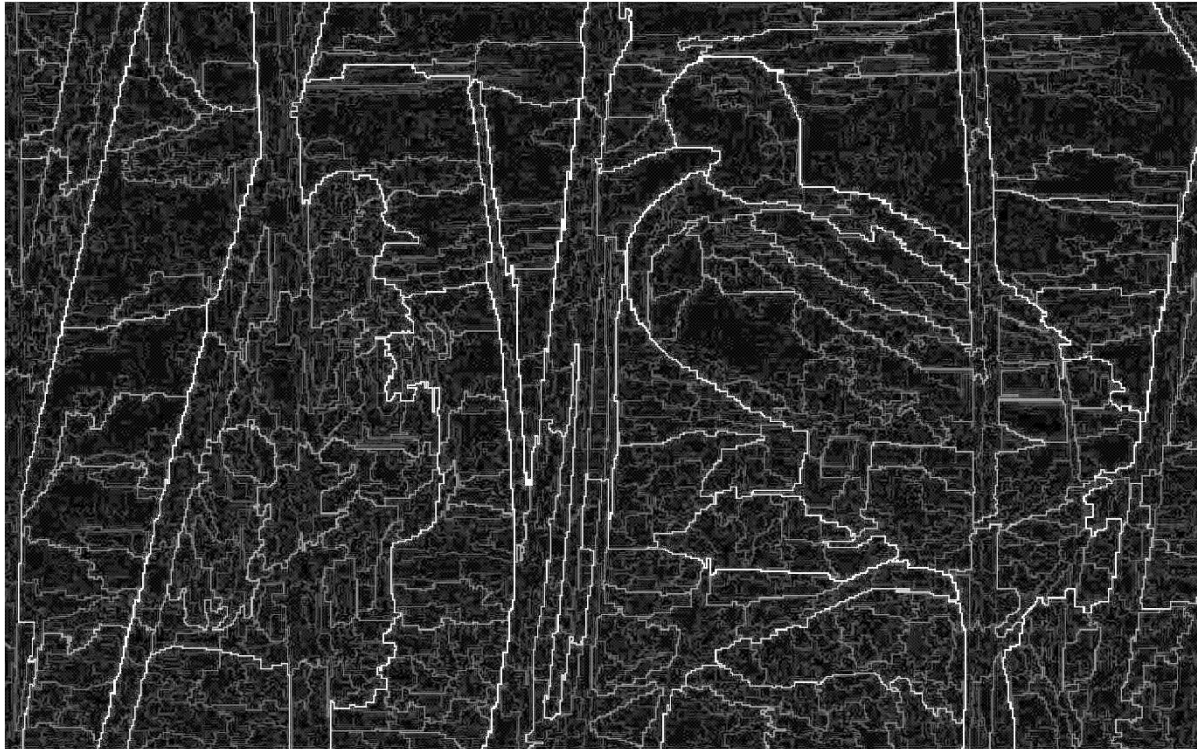
RGB



Honeymoon on the Danube River in Budapest 😊

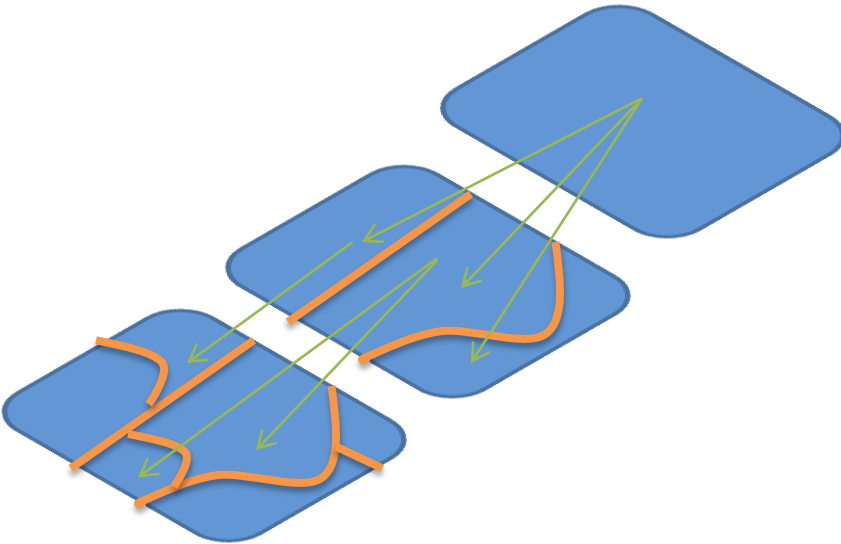
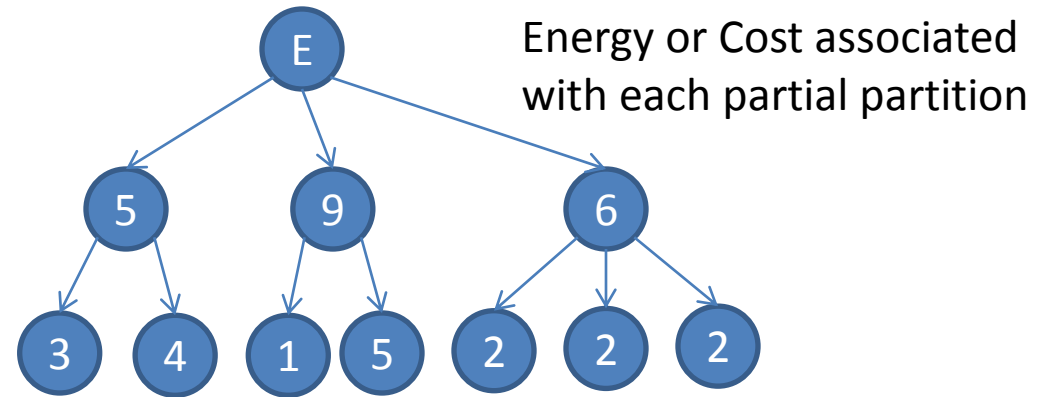
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# Saliency and Hierarchy



Thresholding the Saliency gives the partitions of the different levels of the Hierarchy of Duck Image

# Generating the hierarchy



Hierarchy of segmentations generated by connected area closings of increasing area parameter

























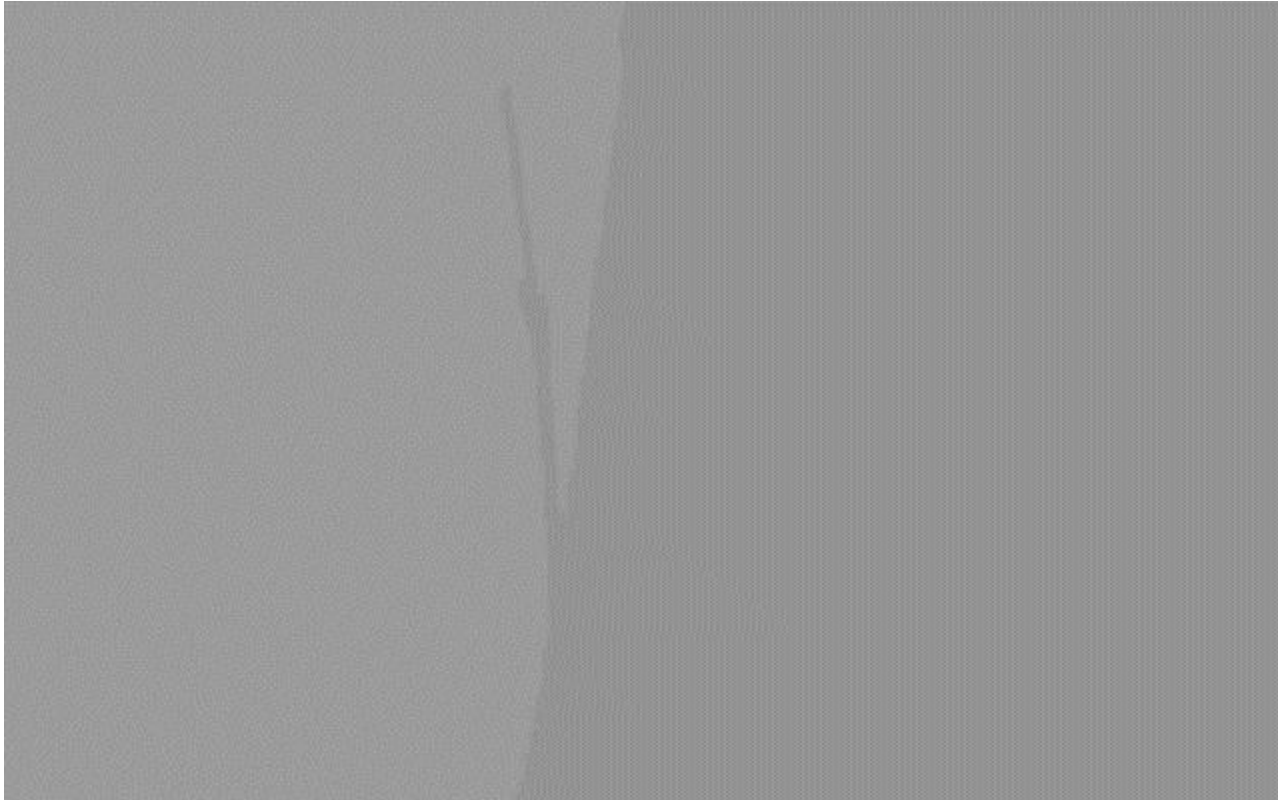


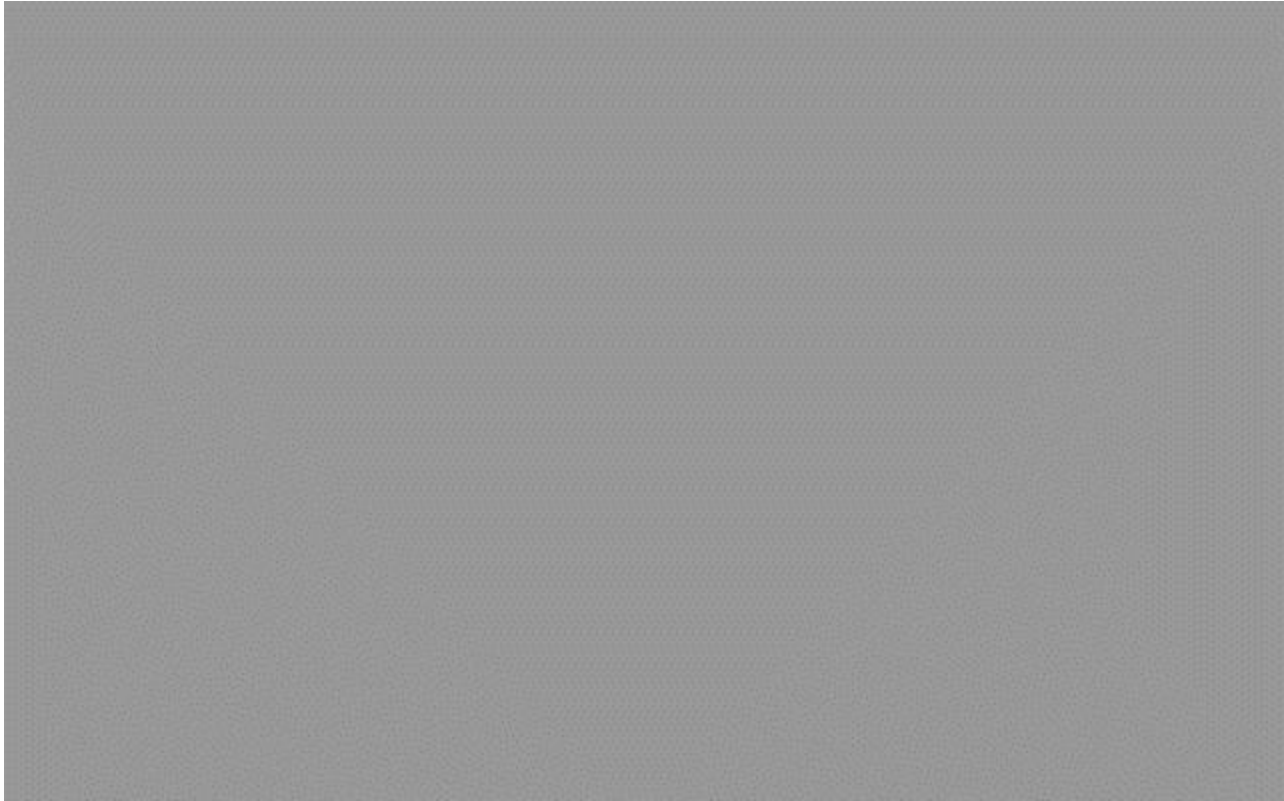






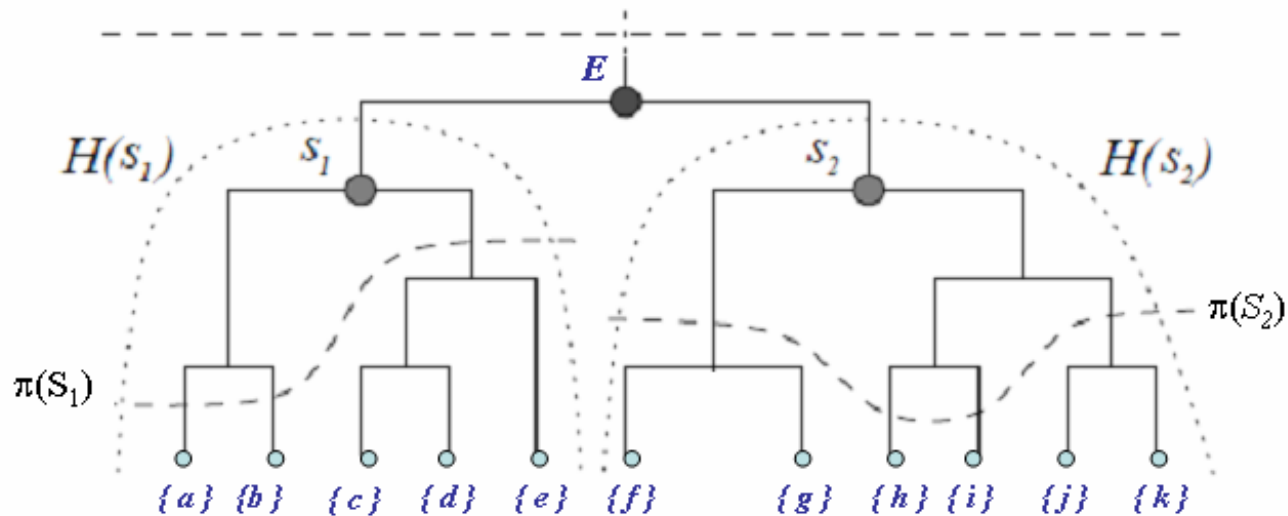






# Cuts

- Given an initial hierarchy  $H$ , a number of partitions of  $E$  whose classes are in  $H$  can be obtained.



- These undulating sections are called **cuts** and denoted by  $\pi$

# Energy and pyramid

There are a combinatorially high number of cuts. To choose among them, we « Climb the pyramid in an principled way ».

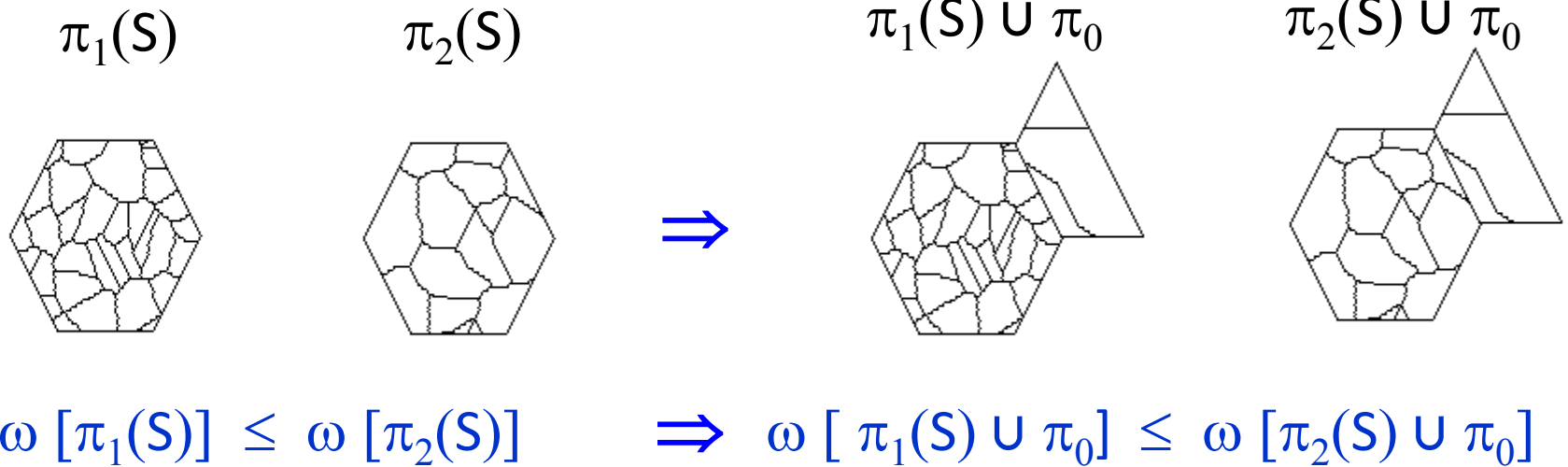
This rests on three **independent** data

- A **pyramid**  $H$  of partitions of  $E$ ,
- A **function**  $f$  on  $E$   
 *$f$  may be the initial image, or another*
- An **energy** i.e. a non-negative function  $\omega$  on the set of all partial partitions.



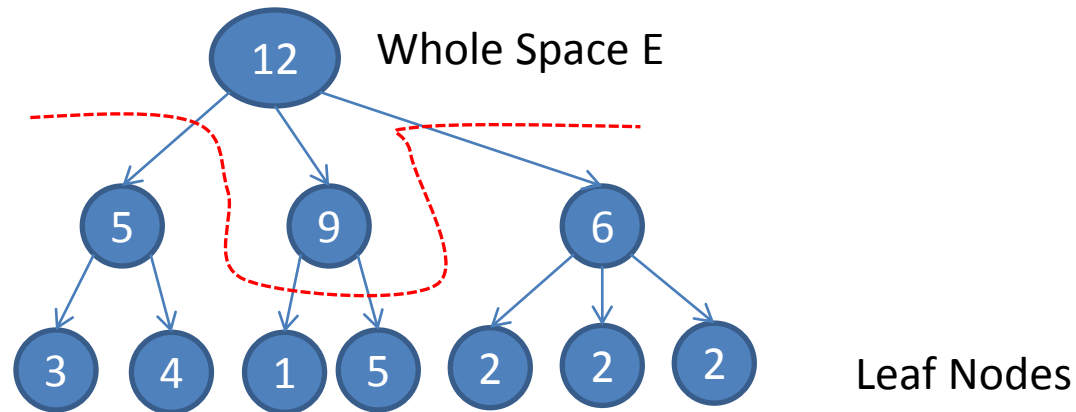
# Hierarchically Increasing Energies

- Is there a cut of *minimum energy* ?
- Answer is Yes in case of *hierarchical increasingness*



At any sub hierarchy the optimal either consists of the summit itself or union of the optimal cut of the sons – **IFF** the energy is H-increasing .

# Optimizing on the hierarchy



$$\omega(\pi(S)) = \sum_{1 \leq u \leq q} \int_{x \in T^u} \|f(x) - m(T^u)\|^2 + \lambda \sum_{1 \leq u \leq q} \left( \frac{k}{2} dT^u + 24 \right)$$

Fidelity Term + (Scale Parameter) Compression Cost

Objective  
function

Lagrange  
Parameter

Constraint

# EXAMPLES

# Optimal Cut



Lambda = 60

Using Average of RGB channels for from the original image to calculate the mean values of classes

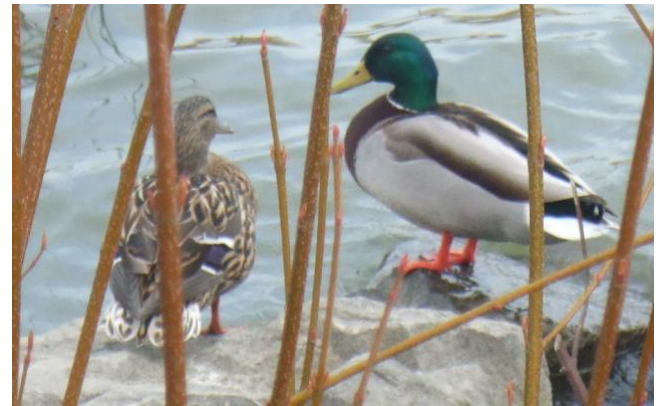
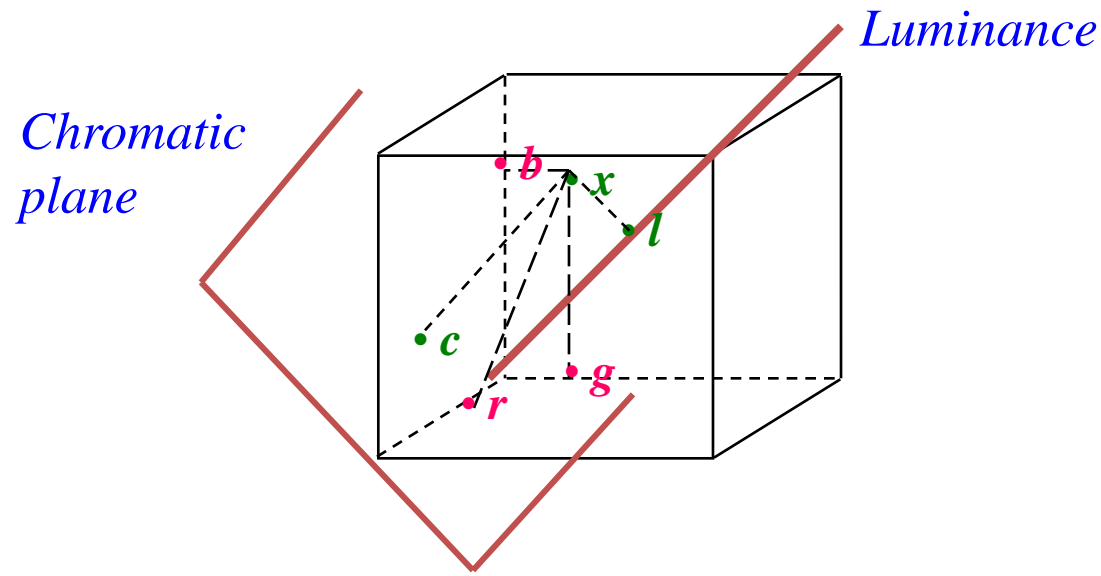
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# Chrominance

Passing to *polar coordinates*, one finds

$$1.5 c \sqrt{2} = (2r - g - b; 2g - b - r; 2b - r - g)$$

$$3l = (r + g + b; r + g + b; r + g + b)$$



# Optimal Cuts

## Luminance Vs Chrominance



Optimal cut  
Lambda = 60

Compression = 21

Number of Classes = 1097

Energy:  $\text{variance}(\text{Luminance}) + \text{perimeter}/2 + \text{coding cost}$

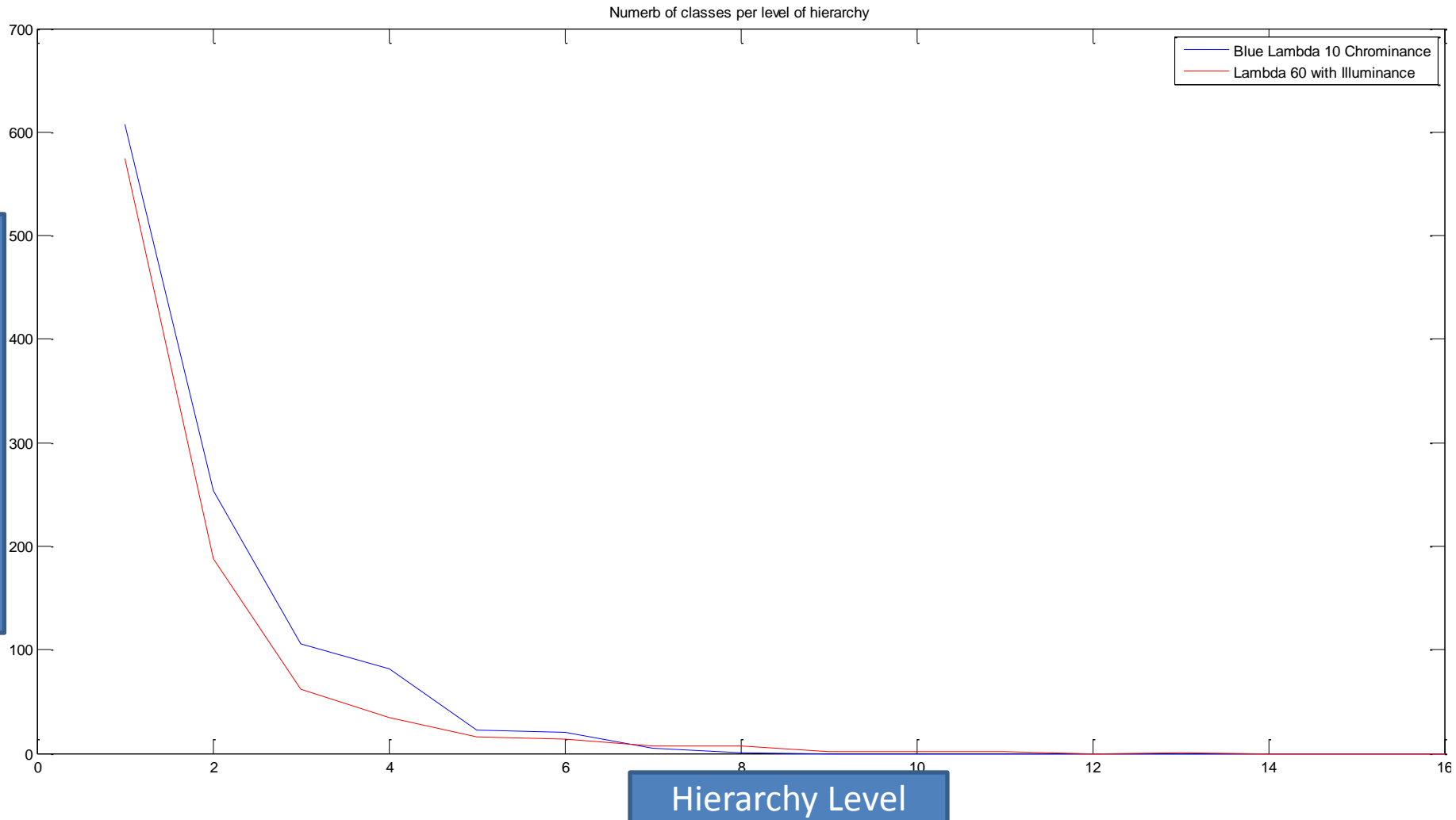
Optimal cut  
Lambda = 10

Compression = 19

Number of Classes = 911

Energy:  $\text{variance}(\text{Chrominance}) + \text{perimeter}/2 + \text{coding cost}$

# Distribution of classes in optimal cuts



The number of classes in the of the best cut using chrominance is

cut = 13, 83

# Future Work

- Developing morphological tools for Geospatial data
- In particular for
  - Energy suitable to model Population
  - Multivariable Climbing
  - Modeling Time evolution of Population, Employment etc.



# Publications and Reports

- Papers
  - Hierarchies and Climbing Energies – Jean Serra, B Ravi Kiran, Jean Cousty: Accepted CIARP 2012
- Technical Reports
  - Climbing the pyramids Technical Report ESIEE, March 2012 – Serra, J., Kiran, B.R. - updated HAL.
  - GIS Datasets and methods of Hierarchical segmentation - Technical Report B Ravi Kiran March 2012
  - Climbing energies and hierarchical Image segmentation for colour images– Technical Report June 2012



# Questions ?